EKA - Linear hypergraphs research group, final report, 2025

Members of the group

- Zoltán Lóránt Nagy (group leader)
- Benedek Kovács (PhD student)
- Dávid R. Szabó (Postdoctoral researcher)

List of papers

Published since the last report

- Benedek Kovács, Zoltán Lóránt Nagy, Dávid R. Szabó, Blocking Planes by Lines in PG(n, q), *Designs, Codes and Cryptography*, (2025), Volume 93, 4403–4432, (Q1)
- Tamás Héger, Z.L. Nagy, Avoiding secants of given size in finite projective planes, *Journal of Combin. Designs* 33(3), 2025, Pages 83-93. (Q2)
- Bence Csajbók, Z.L. Nagy, Complete 3-term arithmetic progression free sets of small size in vector spaces and other abelian groups, *Journal of Combin. Theory, Ser. A.* (2025) 215, 106061 (D1)
- Christian Elsholtz, Jakob Führer, Erik Füredi, Benedek Kovács, Péter Pál Pach, Dániel Simon, Nóra Velich, Maximal line-free sets in $\mathbb{F}_p^n$, *Periodica Mathematica Hungarica*, 7-21., Volume 90 (2025). (Q2)
- Benedek Kovács, Finding a perfect matching of $\mathbb{F}_2^n$ with prescribed differences, *Ars Mathematica Contemporanea*, Accepted Manuscripts (2024). (Q1)
- Benjamin Móricz, Z.L.Nagy, Maximizing the number of rational-value sums or zero-sums, (2026) European Journal of Combinatorics 133, March 2026, 104324. (D1)
- Anurag Bishnoi, Bartłomiej Kielak, Benedek Kovács, Zoltán Lóránt Nagy, Gábor Somlai, Máté Vizer, Zeyu Zheng, The generalized trifference problem, (2025) IEEE Transactions on Information Theory, Vol. 72, No. 5, 2907-2914. (D1)

Submitted

- Gergely Kiss, Ádám Markó, Zoltán Lóránt Nagy, Gábor Somlai, On polynomials of small range sum. (2024)
- Benedek Kovács, Zoltán Lóránt Nagy, Dávid R. Szabó, Settling the no-$(k+1)$-in-line problem when k is not small (2025)
- Benedek Kovács, Zoltán Lóránt Nagy, Dávid R. Szabó, Randomised algebraic constructions for the no-$(k+1)$-in-line problem (2025)
- Dávid R. Szabó: Rational normal curves as no-$(d+2)$-on-Q-quadric sets, (2025) *arXiv preprint*

- Benedek Kovács, Zoltán Lóránt Nagy, Cardinalities of the total number of independent sets (2025)
- Benedek Kovács, Code-based $[3,1]$-avoiders in finite affine spaces AG$(n,2)$ (2025)
- Endre Csóka, Panna Tímea Fekete, Zoltán Lóránt Nagy, Levente Szemerédi, Bisection width, max-cut and internal partitions of 5-regular graphs, (2025)

List of further papers supported by the grant, not listed above

- Benedek Kovács, Zoltán Lóránt Nagy, Avoiding intersections of given size in finite affine spaces AG(n,2), ***Journal of Combinatorial Theory, Series A***, 209, 105959 (2025). (D1)
- Benedek Kovács, Zoltán Lóránt Nagy, Multicolor Turán numbers II. - a generalization of the Ruzsa-Szemerédi theorem and new results on cliques and odd cycles, ***Journal of Graph Theory***, 107(3), 629-641 (2024). (D1)
- János Barát, Andrzej Grzesik, Attila Jung, Zoltán Lóránt Nagy, Dömötör Pálvölgyi, The double Hall property and cycle covers in bipartite graphs, ***Discrete Mathematics***, 347(9), 114079 (2024). (Q1)
- Zoltán Lóránt Nagy, Partitioning the projective plane to two incidence-rich parts, ***Journal of Combinatorial Designs*** 32(12) 703-714. (2024). (Q2)
- Dániel T. Nagy, Zoltán Lóránt Nagy, Russ Woodroofe, The extensible No-Three-In-Line problem, ***European Journal of Combinatorics***, 114, December 2023, 103796 (D1)
- Zsolt Baja, Dániel Dobák, Benedek Kovács, Péter Pál Pach, Donát Pigler, Towards characterizing the 2-Ramsey equations of the form $ax + by = p(z)$, ***Discrete Mathematics***, 346(5), May 2023, 113324 (Q1)

Activities in the final year

- Participation and talks on conferences
  - Random Structures and Algorithms conference, Wien, 2025.08.03-08.08., B. Kovács, Z.L. Nagy - speakers
  - Eurocomb 2025, 08.25-08.29, D. Szabó, Z.L.Nagy - speakers
  - Banff, BIRS workshop on Finite geometry and Ramsey theory, 2025.09.05-09.12., Z.L. Nagy, Zsuzsa Weiner
  - Geometry Workshop 2025 (Szeged) 10.23-10.26., Szeged, D. Szabó - speaker
  - 5th Pythagorean Conference (Kalamata) 2025.06.01-06.06. B. Kovács - speaker
- Workshop organization (Z.L. Nagy)
  18th Emléktábla Workshop on extremal combinatorics.
  location: Velence. Date: 07.21-07.25 (2025). webpage: https://users.renyi.hu/ emlektab/
- Dissemination

  - Problem solving and Research seminar at ELTE Bolyai College (Z.L. Nagy),
  - Math camp programs for 18-year-olds (Z.L.Nagy, B. Kovács),
  - Supervision in extremal combinatorics and additive combinatorics (EKÖP for freshers/ 2nd year student, also at PhD level)
  - REU program, supervision (Z.L.Nagy)
  - support for BSc students to have research experience in related fields (Kitti Kurucz, Artúr Nádor, Dávid Foris, Benjamin Móricz) – all their work resulted in TDK theses.
  - home page for the research group: `https://nagyzoli.web.elte.hu/RG.html`

- Collaboration with researchers from abroad
  (Anurag Bishnoi (TU Delft), Andrzej Grzesik (Jagellonian, Kraków), Nina Kamcev (University of Zagreb), Sam Mattheus (Vrije Universiteit Brussel), Sam Adriaensen (Vrije Universiteit Brussel), Cory Palmer (Uni. Montana),)

- grant applications, prizes
  - Applied for The Momentum Grant (Starting category) - (not successful)
  - received the Bolyai Scholarship (2025)
  - received the Turán prize of the Hungarian Academy of Sciences (2025)

Summary of the main results in the final year

We describe the progress made in several areas, according to the research plan. As presented before, these are in accordance with the publication plan and milestones of the proposal. For the past years, see the details on the homepage of the research group.

During the past year, following previous results, extremal combinatorial problems were studied concerning forbidden configurations in finite affine or projective spaces (or planes). Several such problems have its analogs or roots in graph theory, more precisely, in Turán-type extremal problems. However, different tools (algebraic and additive combinatorial) are also needed beside combinatorial and probabilistic tools.

Results over vector spaces and finite geometries

Together with Anurag Bishnoi, B. Kovács, Gábor Somlai, Zeyu Zheng and Bartlomiej Kielak, we considered the generalisation of the trifference problem. Let Σ denote our alphabet, usually $\Sigma = \{0, 1, 2\}$ or a finite field $\mathbb{F}_q$. If $a, b, c \in \Sigma^n$ are three codewords, then we say that a, b and c *triffer* at position i if $\{a_i, b_i, c_i\}$ is a 3-element set (no repetition occurs).
Let $T(n, m)$ be the largest size of a $C \subseteq \{0, 1, 2\}^n$ such that for any three distinct elements x, y, z in C, we have at least m coordinates where they triffer. $T(n, 1)$ is the well-known trifference problem, which is still wide open, and connected to extensively studied fields in finite geometry as minimal codes and blocking sets. We proved lower and upper bounds on the general function $T(n, m)$, and showed a phase transition w.r.t. its order of magnitude at $m \sim \frac{2}{9}n$.

Together with B. Kovács and D. Szabó, we studied the problem of determining the maximum number $f_k(n)$ of grid points in an $[1, n] \times [1, n]$ grid, such that the point set does not contain $k+1$ collinear points. This is a natural generalisation of one of the most

well-known problems in combinatorial geometry, the no-3-in-line problem. We proved that the precise answer is kn, provided that $k > C(n \log n)^{1/2}$ for an absolute constant C. The proof relies on carefully constructed bi-uniform random bipartite graphs and concentration inequalities.

We showed that $\left(1 - \frac{2}{k}\right) kn \leq f_k(n) \leq kn$ and $\left(1 - \frac{3}{k}\right) kn \leq f_k(n) \leq kn$ hold for every even k and odd k, respectively, provided that n is large enough. This is asymptotically tight as $k \to \infty$. Previously, only $f_k(n) = \Omega(kn)$ was known due to Lefmann. We presented further improvements on the lower bounds for constant values of k when $k < 23$ holds. All these bounds are based on randomised algebraic constructions.

D. Szabó studied the maximal number $g_d(n)$ of points from $[1, n]^d$ such that no Euclidean hyperplane contains $d+1$ selected points and no hypersphere contains $d+2$ selected points. He showed $n - o(n) \leq g_d(n) \leq dn$ (the best known bounds) using tools from algebraic geometry over finite fields. He generalised this result to other quadric hypersurfaces.

A set of points in $\mathrm{AG}(n, 2)$ is called $[k, t]$-avoiding if there is no k-dimensional affine subspace containing exactly t points of S. Together with B. Kovács, we made the following conjecture in our recent paper "Avoiding intersections of given size in finite affine spaces $AG(n, 2)$" at *J.Combin. Theory, Ser. A*: the density of sizes $|S|$ for which S can be $[k, t]$-avoiding tends to zero, and proved this result in the cases $t \in \{0, 2^{k-1}, 2^k\}$. This year, in a new work of B. Kovács, a large family of explicit $[k, 1]$-avoiding sets is presented, giving exponentially many different sizes. The construction uses weight enumerator polynomials of binary linear codes to determine $|S|$, and makes use of group-theoretical tools to ensure distinct cardinalities.

Extremal combinatorics

In a new paper, we studied the set of numbers the total number of independent sets can admit in n-vertex graphs. We proved that the cardinality of this set $Ni(n)$ is very close to 2^n in the following sense: $Ni(n)/2^n = O(n^{-1/5})$ while for infinitely many n, we have $\log_2(Ni(n)/2^n) < -2^{(1+o(1))\sqrt{\log_2 n}}$. This set is also precisely the set of possible values of the independence polynomial $I_G(x)$ at $x = 1$ for n-vertex graphs G. As an application, we give further improved constructions of $[k, 1]$-avoiding sets in $\mathbb{F}_2^n$. This was a joint work with B. Kovács.

In the paper entitled "Bisection width, max-cut and internal partitions of 5-regular graphs, joint with Endre Csóka, Panna Tímea Fekete, and Levente Szemerédi, we presented a new factor of IID process based on the local algorithm introduced by Díaz, Serna, and Wormald (2007). This new approach allows us to improve the previously known upper bounds on the minimum and maximum bisection width and the maximum cut of random d-regular graphs for $d > 4$ by introducing a new recoloring phase after the termination of the original algorithm. As an application, we showed that random 5-regular graphs asymptotically almost surely admit an internal partition, i.e., a partition of the vertex set into two non-empty classes so that every vertex has at least half of its neighbours in its own class.

Results not yet announced

- In a recent research collaboration of B. Kovács with A. Bishnoi at TU Delft (October

10-21, 2025) we vastly improved the best-known lower bounds on the sizes of r-*bounded* trifferent codes (i.e. those which have exactly r digits 2 in each codeword). A result of Bhandari and Khetan showed that there exist such codes of length n and size $\Omega(n^{r^{0.36}})$. Our improved bound gives a size of $n^{\lceil r/2 \rceil - o(1)}$. In our concatenation technique, the underlying r-uniform hypergraph is given by a construction of Alon and Shapira, avoiding the $(2r, 3)$-configuration (meaning 3 hyperedges on $\leq 2r$ vertices), which is a generalization of the famous Ruzsa-Szemerédi $(6, 3)$-problem.

- In a research project offered to students, we studied the following problem: What is the maximum number of points one can select from the $n \times n$ square grid that avoids forming certain forbidden configurations? Several cases of this problem have been studied earlier, with numerous influential results. With A. Nádor and M. Jánosik, we focused on the case of rhombuses, parallelograms, squares and kites and proved bounds using the combination of additive combinatorics and probabilistic techniques.

- A set of points $S \subseteq \mathbb{F}_p^n$ is called *p-divisible* if every affine hyperplane in $\mathbb{F}_p^n$ intersects S in a number of points congruent to 0 (mod p). The Strong Cylinder Conjecture of Ball asserts that if S is a p-divisible set of p^2 points in $\mathbb{F}_p^3$, where p is a prime, then S is a cylinder. Together with Á. Markó, G. Kiss and G. Somlai, we studied the cylinder conjecture and proved some partial (structural) results, namely we showed that every S is a $\mathbb{Z}$-linear combination of cylinders, provided that the conditions of the cylinder conjecture hold.

Zoltán Lóránt Nagy